UW-Whitewater / UT-Dallas Mathematics Meet – April 2006
MIDDLE SCHOOL

Problem 1

Michael is celebrating his fifteenth birthday today. How many Sundays have there been in his lifetime?

Problem 2

Evaluate $\dfrac{\dfrac{1}{\dfrac{1}{10}-\dfrac{1}{12}}}{\dfrac{1}{\dfrac{1}{8}-\dfrac{1}{6}}+\dfrac{1}{\dfrac{1}{5}-\dfrac{1}{6}}}$.

Problem 3

Find the sum of all the prime numbers less than 100 which are one more than a multiple of six.

Problem 4

At the beginning of each hour from 1 o'clock AM to 12 NOON and from 1 o'clock PM to 12 MIDNIGHT a coo-coo clock's coo-coo bird coo-coos the number of times equal to the number of the hour. In addition, the coo-coo clock's coo-coo bird coo-coos a single time at 30 minutes past each hour. How many times does the coo-coo bird coo-coo from 12:42 PM on Monday until 3:42 AM on Wednesday?

Problem 5

The sizes of the freshmen class and the sophomore class are in the ratio 5:4. The sizes of the sophomore class and the junior class are in the ratio 7:8. The sizes of the junior class and the senior class are in the ratio 9:7. If these four classes together have a total of 2158 students, how many of the students are freshmen?

Problem 6

We draw a radius of a circle. We draw a second radius 23 degrees clockwise from the first radius. We draw a third radius 23 degrees clockwise from the second. This continues until we have drawn 40 radii each 23 degrees clockwise from the one before it. What is the measure in degrees of the smallest angle between any two of these 40 radii?

Problem 7

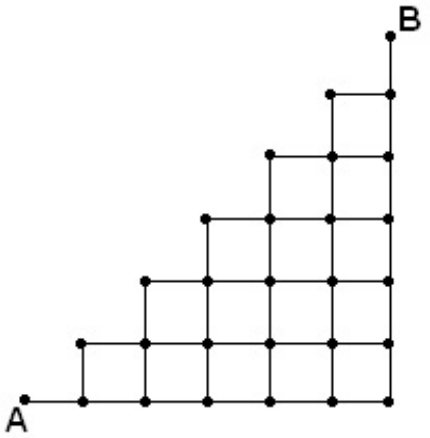
At a movie theater tickets for adults cost 4 dollars more than tickets for children. One afternoon the theater sold 100 more child tickets than adult tickets for a total sales amount of 1475 dollars. How much money would the theater have taken in if the same tickets were sold, but the costs of the child tickets and adult tickets were reversed?

Problem 8

A rogue spaceship escapes. 54 minutes later the police leave in a spaceship in hot pursuit. If the police spaceship travels 12% faster than the rogue spaceship along the same route, how many minutes will it take for the police to catch up with the rogues?

Problem 9

Moving horizontally and vertically from point to point along the lines in the diagram below, how many routes are there from point A to point B which consist of six horizontal moves and six vertical moves?



Problem 10

How many rectangles are there in the diagram below such that the sum of the numbers within the rectangle is a multiple of 7?

43	44	45	46	47	48	49
36	37	38	39	40	41	42
29	30	31	32	33	34	35
22	23	24	25	26	27	28
15	16	17	18	19	20	21
8	9	10	11	12	13	14
1	2	3	4	5	6	7

Problem 11

Consider the polynomials

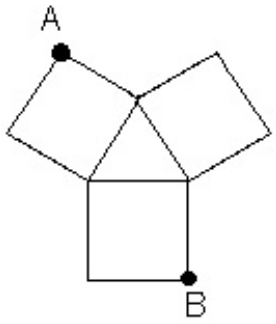
$$P(x) = \left(x + \sqrt{2}\right)\left(x^2 - 2x + 2\right)$$
$$Q(x) = \left(x - \sqrt{2}\right)\left(x^2 + 2x + 2\right)$$
$$R(x) = \left(x^2 + 2\right)\left(x^8 + 16\right).$$

Find the coefficient of x^4 in $P(x) \cdot Q(x) \cdot R(x)$.

Problem 12

How many positive integers divide the number 10! = 1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 ?

Problem 13



An equilateral triangle with side length 6 has a square of side length 6 attached to each of its edges as shown. The distance between the two farthest vertices of this figure (marked A and B in the figure) can be written as $m + \sqrt{n}$ where *m* and *n* are positive integers.
Find $m + n$.

Problem 14

The rodent control task force went into the woods one day and caught 200 rabbits and 18 squirrels. The next day they went into the woods and caught 3 fewer rabbits and two more squirrels than the day before. Each day they went into the woods and caught 3 fewer rabbits and two more squirrels than the day before. This continued through the day when they caught more squirrels than rabbits. Up through that day how many rabbits did they catch in all?

Problem 15

A concrete sewer pipe fitting is shaped like a cylinder with diameter 48 with a cone on top. A cylindrical hole of diameter 30 is bored all the way through the center of the fitting as shown. The cylindrical portion has height 60 while the conical top portion has height 20. Find N such that the volume of the concrete is $N\pi$.