Spring 2004 UW-Whitewater Middle School Mathematics Meet

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Instructions

Teams may fill in answers to the questions over the next 90 minutes. At any time contestants can click the SUBMIT button to submit their team's entry. Answers may be submitted multiple times by the same team, but only the last set of answers received before the contest ends will be accepted and graded. Contestants are allowed to work on these problems as a team. No help may be provided by persons not on their team. There is no penalty for guessing. (See official rules for complete contest rules.)

Problem 1

This year February 29 fell on a Sunday. In what year will February 29 next fall on a Sunday?

Problem 2

If $h(a,b,c) = \dfrac{abc}{a+b+c}$, find $h(3\sqrt{5}, 6\sqrt{5}, 9\sqrt{5})$.

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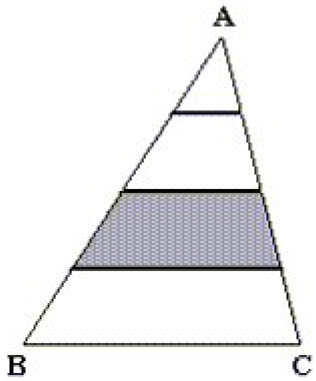
Problem 3

In triangle ABC, three lines are drawn parallel to side BC dividing the altitude of the triangle into four equal parts. If the area of the second largest part is 35, what is the area of the whole triangle ABC ?



Problem 4

If the numbers $2a + 2$ and $2b + 2$ add up to 2004, find the sum of the numbers $\frac{a}{2} - 2$ and $\frac{b}{2} - 2$.

Problem 5

Write the number $2004_{(5)}$ [2004 base 5] as a number in base 6.

Problem 6

How many different positive integers divide 10! ?

Problem 7

A rectangle has area 1100. If the length is increased by ten percent and the width is decreased by ten percent, what is the area of the new rectangle?

Problem 8

The number 2.5081081081081... can be written as m/n where m and n are natural numbers with no common factors. Find $m + n$.

Problem 9

How many positive integers less that 200 are relatively prime to either 15 or 24?

Problem 10

One rainy afternoon you write the number 1 once, the number 2 twice, the number 3 three times, and so forth until you have written the number 99 ninety-nine times. What is the 2005th digit that you write?

Problem 11

Find the sum of all integers x satisfying $1 + 8x \leq 358 - 2x \leq 6x + 94$.

Problem 12

If $f(x, y) = xy + 2x + y + 1$, find $f(f(2, f(3, 4)), 5)$.

Problem 13

How many three digit numbers are made up of three distinct digits?

Problem 14

A polygon has five times as many diagonals as it has sides. How many vertices does the polygon have?

Problem 15

Find the prime number p for which $p + 2500$ is a perfect square.

Problem 16

A week ago, Sandy's seasonal Little League batting average was 360. After five more at bats this week, Sandy's batting average is up to 400. What is the smallest number of hits that Sandy could have had this season?

Problem 17

Find x so that $2^{2^{3^{2^{2}}}} = 4^{4^{x}}$.

Problem 18

Find the number of addition problems in which a two digit number is added to a second two digit number to give a two digit answer, such as in the three examples:

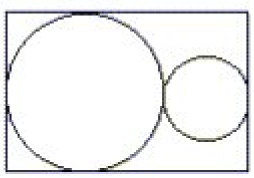
$$\begin{array}{c}23\\ \underline{42}\\ 65\end{array}, \quad \begin{array}{c}36\\ \underline{36}\\ 72\end{array}, \quad \begin{array}{c}42\\ \underline{23}\\ 65\end{array}.$$

Problem 19

Find n such that $n - 76$ and $n + 76$ are both cubes of positive integers.

Problem 20

A circle with area 40 is tangent to a circle with area 10. Let R be the smallest rectangle containing both circles. The area of R is $\frac{n}{\pi}$. Find n.



Problem 21

Find the number of different quadruples (a, b, c, d) of positive integers such that $ab = cd = a + b + c + d - 3$.

Problem 22

Two circles have radii 15 and 95. If the two external tangents to the circles intersect at 60 degrees, how far apart are the centers of the circles?

Problem 23

A cubic block with dimensions n by n by n is made up of a collection of 1 by 1 by 1 unit cubes. What is the smallest value of n so that if the outer layer of unit cubes are removed from the block, more than half the original unit cubes will still remain?

Problem 24

Let a be a real number greater than 1 such that $\dfrac{20a}{a^2+1} = \sqrt{2}$.
Find $\dfrac{14a}{a^2-1}$.

Problem 25

In the addition problem

```
  W H I T E
+ W A T E R
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P I C N I C
```

each distinct letter represents a different digit. Find the number represented by the answer `PICNIC`.