Problem 1
Penelope is 36 years old. She noticed that the sum of her age and her father’s age is 5 times the difference in her age and her father’s age. Find Penelope’s father’s age.

Problem 2
The diagram below shows an $11 \times 13$ rectangle and a $11 \times 21$ rectangle attached to adjacent sides of an $11 \times 11$ square. Find the distance between the two farthest apart points in this figure.

![Diagram](image)

Problem 3
Fred placed 19 blue marks on a pole that divided the pole into 20 equally-sized sections. Karen placed 16 red marks on the pole that divided the pole into 17 equally-sized sections. The distance between adjacent blue marks is $m$ percent less than the distance between adjacent red marks, where $m$ is a positive integer. Find $m$.

Problem 4
The diagram below shows a large equilateral triangle with side length 8 divided into 16 small equilateral triangles with side length 2. Find the total length of all the line segments in the diagram.
Problem 5
Let $a$ and $b$ be nonzero real numbers such that

$$(a - 10b)^2 + (a - 11b)^2 + (a - 12b)^2 = (a - 13b)^2 + (a - 14b)^2.$$ 

Find $\frac{a}{b}$.

Problem 6
Find the difference between the base-seven number $234_7$ and the base-six number $234_6$. Express the answer as a base-ten number.

Problem 7
Let $ABCD$ be a square with side length 24, and let $E$ and $F$ be the midpoints of sides $AB$ and $CD$, respectively. Find the area of the region common to the insides of both $\triangle ABE$ and $\triangle CDE$.

Problem 8
Find the positive integer $n$ such that

$$\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18} + \frac{1}{24} + \frac{1}{42} + \frac{1}{n} = 1.$$ 

Problem 9
In $\triangle ABC$ with right angle at $C$, points $D$ and $E$ lie on side $AB$ and $AC$, respectively, such that $CD$ is an altitude of $\triangle ABC$ and $DE$ is an altitude of $\triangle ACD$. Suppose $CD = 10$ and $DE = 8$. Then the area of $\triangle ABC$ is $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

Problem 10
Nonnegative integers $m$ and $n$ satisfy $46^m - 2 \cdot 46^n = 2024$. Find $46^n + 2 \cdot 46^m$.

Problem 11
Find the positive integer $n$ such that there is an integer $b > 1$ where the base-$b$ representation of $n$ is 961 and the base-$(b + 1)$ representation of $n$ is 804.

Problem 12
Find the sum of the squares of all integers $n$ for which $(n + 9)^2$ divides the positive integer $n + 2024$.

Problem 13
For any real number $y$, let $\{y\}$ refer to the fractional part of $y$, so, for example, $\{3.14\} = 3.14 - 3 = 0.14$, $\{10\} = 10 - 10 = 0$, and $\{-2.7\} = -2.7 - (-3) = 0.3$. Suppose $x$ satisfies $3x + \{x\} = 100$. Find $4x$. 

2
Problem 14
In the following arithmetic calculation, each different letter represents a different digit:

\[ PU \underline{R} + P \underline{L}E - C \underline{OM}E \underline{T} + M \underline{E}E \underline{T} = 0. \]

Find the minimum possible value for the four-digit number \( M \underline{E}E \underline{T} \).

Problem 15
In rectangle \( ABCD \), \( AB = 20 \) and \( AD = 19 \). Point \( E \) lies on side \( AD \) with \( AE = 4 \). Let the incircle of \( \triangle CDE \) be tangent to \( CE \) at \( F \). A circle tangent to \( AB \) and \( BC \) is tangent to \( CE \) at \( G \). The distance \( FG \) can be written \( \frac{m}{n} \), where \( m \) and \( n \) are relatively prime positive integers. Find \( m + n \).

Problem 16
Three red blocks, three white blocks, and three blue blocks are packed away by randomly selecting three of the nine blocks to go into a red box, then randomly selecting three of the six remaining blocks to go into a white box, and then placing the remaining three blocks in a blue box. The probability that no red blocks end up in the red box and no white blocks end up in the white box is \( \frac{m}{n} \), where \( m \) and \( n \) are relatively prime positive integers. Find \( m + n \).

Problem 17
The least real number \( r \) such that \( 2x + 3y + 4z \leq 3x^2 + 4y^2 + 12z^2 + r \) for all real numbers \( x, y, \) and \( z \) is a rational number \( \frac{m}{n} \), where \( m \) and \( n \) are relatively prime positive integers. Find \( m + n \).

Problem 18
Find the number of ordered pairs \((A, B)\) of sets satisfying \( A \subseteq B \subseteq \{1, 2, 3, 4, 5, 6\} \) where the number of elements in \( A \) plus the number of elements in \( B \) is an even number.

Problem 19
An isosceles triangle \( R \) has side lengths 17, 17, and 16. Region \( S \) consists of the set of points inside of \( R \) that are a distance of at least 2 from the sides of \( R \). The area of \( S \) divided by the area of \( R \) is \( \frac{m}{n} \), where \( m \) and \( n \) are relatively prime positive integers. Find \( m + n \).

Problem 20
A 25 meter pipe that connects two reservoirs is made up of alternating 2-meter sections and 3-meter sections: 2, 3, 2, 3, 2, 3, 2, 3, 2, 3. Suppose three of these ten sections are selected at random and removed from the pipe. Then there are relatively prime positive integers \( m \) and \( n \) such that \( \frac{m}{n} \) is the probability that the three sections can be reinserted into the pipe in a way that none of the three sections ends up in the position where it started and none of the other seven sections of pipe are moved. The 2-meter and 3-meter sections need not be alternating in the new arrangement. Find \( m + n \).