PURPLE COMET! MATH MEET April 2026

MIDDLE SCHOOL - PROBLEMS

Problem 1

One angle in a hexagon measures 90°. The other angles all measure x degrees. Find x.

Problem 2

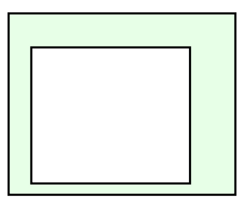
On Monday, a box of donuts is discounted by 10 percent. On Tuesday, that lower price has a 20 percent discount applied. Find the overall percentage of discount.

Problem 3

Find the number of positive integers less than or equal to 2026 with the property that each of its digits are either a 0, 2, or 6.

Problem 4

The outside rectangle in the diagram below has horizontal dimension 20 and vertical dimension 16. The inside rectangle is a distance 1 from the bottom, a distance 2 from the left, a distance 3 from the top, and a distance 4 from the right of the outside rectangle. Find the area of the shaded region between the two rectangles.



Problem 5

Find the number of integers x that satisfy $x^2 + 5x - 204 < 0$.

Problem 6

Find the value of n that satisfies the following equation:

$$\frac{2026n + 2027n}{2028n - 2025n} = 2029n - 2024n + 1.$$

Problem 7

For certain (not necessarily distinct) prime numbers $q_1, q_2, q_3, \ldots, q_7$, the number $q_1^2 + q_2^2 + q_3^2 + \cdots + q_7^2$ is an odd perfect square. Find $q_1 + q_2 + q_3 + \cdots + q_7$.

Problem 8

A marching band 100 meters long is marching forward down the street at a constant speed of 2 kilometers per hour. The band's color guard carrying a flag at the back of the band starts to walk toward the front of the band at a constant speed of 4 kilometers per hour. When she reaches the front of the band, she turns around and walks back at the same constant speed toward the back of the band. By the time the color guard reaches the back of the band, she has walked a total of $\frac{m}{n}$ meters, where m and n are relatively prime positive integers. Find $m + n$.

Problem 9

A cube with edge length 10 is made up of 1000 cubes of edge length 1, all of whose sides are white. Suppose that the entire outside surface of the large cube is painted green so, as a result, some of the sides of some of the 1000 small cubes get painted green. Each of the 1000 small cubes is then individually rolled so that a random side faces up. Find the expected number of the 1000 sides facing up that are painted green.

Problem 10

On a test, the mean score of the students who passed was 76, the mean score of the students who failed was 42, and the mean score of the whole class was 60. The fraction of the class who passed can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Problem 11

Lazar picks a four-digit positive integer at random. The probability that exactly two of the digits in the number Lazar picks are prime numbers is $\frac{m}{n}$, where m and n are relatively prime positive integers.
Find $m + n$.

Problem 12

Find the maximum value of $a(1 - a) + b(3 - b) + c(5 - c) + d(7 - d)$, where a, b, c, and d are real numbers.

Problem 13

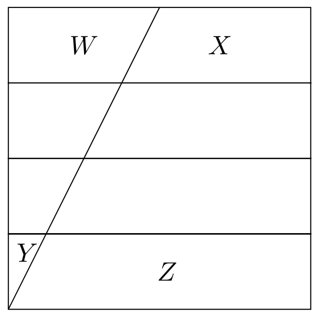
Casey, Pat, and Sandy looked at a list of all the subsets of the set S. Casey circled all of the subsets that contained exactly 2 elements, Pat circled all of the subsets that contained exactly 8 elements, and Sandy circled all of the subsets that contained exactly 13 elements. Pat and Sandy both circled the same positive number of subsets. Find the number of subsets Casey circled.

Problem 14

Let $a_1 = -2$, $a_2 = -1$, and $a_{n+1} = \sqrt[3]{n(a_n^2 + 1) + 2a_{n-1}}$, for all $n \geq 2$. Find a_{2026}.

Problem 15

A square is divided into four congruent rectangles, and then a line is drawn from one vertex further dividing the four rectangles, as shown below. The ratio of the area of region W to the area of region X is $1 : 3$. Find n so that the ratio of the area of region Y to the area of region Z is $1 : n$.

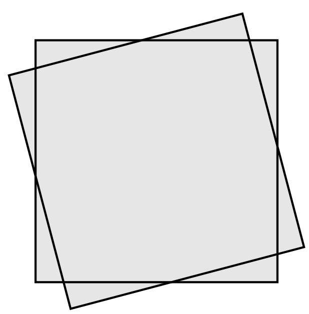


Problem 16

Set A contains 25 positive integers whose mean is 50. The set B contains 36 positive integers whose mean is 70. Find the minimum possible mean of the numbers in the set $A \cup B$.

Problem 17

Let S be a square with side length 16. Let S' be S rotated around the center of S so that the region enclosed by the union of the two squares has area 288. The intersection of the insides of the two squares is an octagon. Find the perimeter of that octagon.



Problem 18

Find the maximum possible value of $m + n$, where m and n are integers such that the quadratic polynomial $10x^2 + mx + n$ has rational roots x_1 and x_2 satisfying $(x_1 + 1)(x_2 + 1) = 1$.

Problem 19

Five red plates, five white plates, and five blue plates are placed in a circle in random order. The probability that no two red plates are adjacent is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Problem 20

Let $ABCD$ be a square with side length 6 in a horizontal plane. Let E and F be the midpoints of sides $\overline{AB}$ and $\overline{CD}$, respectively. Let G and H be points above the plane so that $\overline{EG}$ and $\overline{FH}$ are perpendicular to the plane, $EG = 5$, and $FH = 3$. Let P be the pyramid with base $ABCD$ and apex G, and let Q be the pyramid with base $ABCD$ and apex H, as shown. The volume of the intersection of P and Q can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.