

PURPLE COMET! MATH MEET April 2025

MIDDLE SCHOOL - PROBLEMS

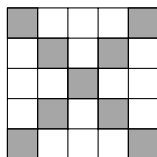
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Problem 1

The first 20 terms of a sequence are 1, 2, 3, 4, 5, 6, 7, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 7, 6, where the terms keep going back and forth between 1 and 7. Find the 2025th term of the sequence.

Problem 2

The 5×5 grid of squares shown below has its diagonal squares shaded grey. The total area of the shaded squares is 18. Find the total area of the white squares that are not shaded.

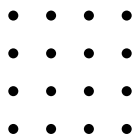


Problem 3

Find the number of positive integers less than 10,000 that contain exactly two digits equal to 2 and exactly one digit equal to 5, such as 2025.

Problem 4

Find the number of rectangles (including squares) whose four vertices are four distinct points in the following 4×4 grid.



Problem 5

Tarisa ran at a constant speed to complete a 12-kilometer route in 80 minutes. Pam rode her bike along the same route going 6 kilometers per hour faster than Tarisa ran. Find the number of minutes it took Pam to complete the 12 kilometers.

Problem 6

Find the least ten-digit positive integer such that the product of its digits is $10!$.

Problem 7

The arithmetic mean of the real numbers a , b , and c is 22. Find the arithmetic mean of the numbers $ab + \frac{c^2}{2}$, $bc + \frac{a^2}{2}$, and $ca + \frac{b^2}{2}$.

Problem 8

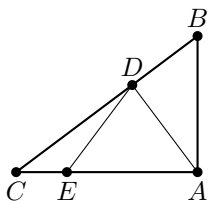
Let p , q , and r be prime numbers such that

$$pqr + 2025 = 3(pq + qr + rp).$$

Find $p + q + r$.

Problem 9

Right triangle $\triangle ABC$ has sides $AB = 75$, $AC = 100$, and $BC = 125$. Point D lies on \overline{BC} and point E lies on \overline{AC} such that $\overline{AD} \perp \overline{BC}$ and $AD = DE$. Find the area of $\triangle ADE$.



Problem 10

The equation $\mathbf{GEO} - \mathbf{MET} + \mathbf{RY} = \mathbf{0}$ shows that zero equals a three-digit integer subtracted from a three-digit integer plus a two-digit integer, where each different letter represents a distinct decimal digit. Find the maximum possible value of the eight-digit number GEOMETRY.

Problem 11

Find the greatest positive integer n such that $n!$ is not divisible by 7^{72} .

Problem 12

Find the positive real number x for which

$$2\sqrt[3]{1 + \frac{x}{2}} + \sqrt[3]{1 - x} = 3.$$

Problem 13

There is a positive real number r such that the combined areas of four circles with radii r , $3r$, $5r$, and $7r$ is 189. Find the difference between the areas of the largest and the smallest of the four circles.

Problem 14

Find the greatest positive integer n such that $(n^2 - n + 1)^2 + n(n - 1)^2$ divides $n^5 - 2100$.

Problem 15

Twelve cards are numbered 1 to 12. Three of these cards are selected at random without replacement. The probability that the three cards can be placed in some order so that their numbers form an arithmetic sequence is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Problem 16

Let x , y , and z be real numbers. Then the maximum possible value of

$$(x + 1)(4y + 1) + (2y + 1)(6z + 1) + (3z + 1)(2x + 1) - (x + 2y + 3z)^2$$

can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $10m + n$.

Problem 17

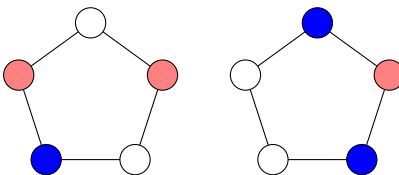
In 3-dimensional coordinate space let $A = (0, 0, 0)$ and $B = (18, 24, 30)$. Find the number of points on the line segment \overline{AB} with the property that exactly 1 of the 3 coordinates of the point is integer valued, such as $(0.75, 1, 1.25)$.

Problem 18

There are positive integers m and n such that $m - n = 18$ and $\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{m} + \frac{1}{n} = \frac{1}{2}$. Find $m + n$.

Problem 19

The ten vertices of two disjoint pentagons are randomly colored so that there are 3 red vertices, 4 white vertices, and 3 blue vertices. The probability that no side of either pentagon connects two red vertices or two blue vertices is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



Problem 20

A rectangular solid measures $12 \times 16 \times 20$. Let A and B be the opposite 12×16 rectangular faces. Sphere S passes through the four vertices of face A and the midpoint of face B . Find the total length of the parts of the edges of the rectangular solid that lie inside sphere S .