

PURPLE COMET! MATH MEET April 2017

MIDDLE SCHOOL - PROBLEMS

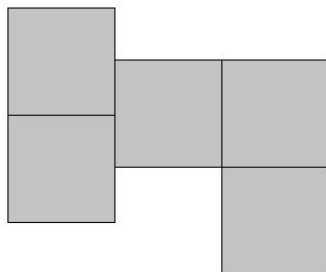
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Problem 1

Caden, Zoe, Noah, and Sophia shared a pizza. Caden ate 20 percent of the pizza. Zoe ate 50 percent more of the pizza than Caden ate. Noah ate 50 percent more of the pizza than Zoe ate, and Sophia ate the rest of the pizza. Find the percentage of the pizza that Sophia ate.

Problem 2

The figure below was made by gluing together 5 non-overlapping congruent squares. The figure has area 45. Find the perimeter of the figure.

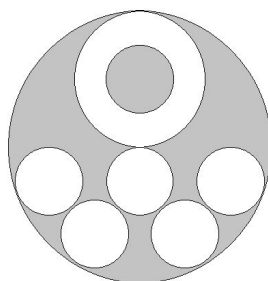


Problem 3

The Stromquist Comet is visible every 61 years. If the comet is visible in 2017, what is the next leap year when the comet will be visible?

Problem 4

The following diagram includes six circles with radius 4, one circle with radius 8, and one circle with radius 16. The area of the shaded region is $k\pi$. Find k .



Problem 5

Find the greatest odd divisor of 160^3 .

Problem 6

On a typical morning Aiden gets out of bed, goes through his morning preparation, rides the bus, and walks from the bus stop to work arriving at work 120 minutes after getting out of bed. One morning Aiden got out of bed late, so he rushed through his morning preparation getting onto the bus in half the usual time, the bus ride took 25 percent longer than usual, and he ran from the bus stop to work in half the usual time it takes him to walk arriving at work 96 minutes after he got out of bed. The next morning Aiden got out of bed extra early, leisurely went through his morning preparation taking 25 percent longer than usual to get onto the bus, his bus ride took 25 percent less time than usual, and he walked slowly from the bus stop to work taking 25 percent longer than usual. How many minutes after Aiden got out of bed did he arrive at work that day?

Problem 7

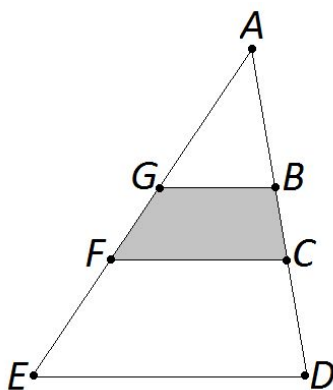
Find the number of positive integers less than 100 that are divisors of 300.

Problem 8

The positive integer m is a multiple of 111, and the positive integer n is a multiple of 31. Their sum is 2017. Find $n - m$.

Problem 9

In $\triangle ADE$ points B and C are on side $\overline{AD}$ and points F and G are on side $\overline{AE}$ so that $\overline{BG} \parallel \overline{CF} \parallel \overline{DE}$, as shown. The area of $\triangle ABG$ is 36, the area of trapezoid $CFED$ is 144, and $AB = CD$. Find the area of trapezoid $BGFC$.



Problem 10

Find the number of rearrangements of the letters in the word **MATHMEET** that begin and end with the same letter such as **TAMEMHET**.

Problem 11

Find the greatest prime divisor of $29! + 33!$.

Problem 12

Let x , y , and z be real numbers such that

$$12x - 9y^2 = 7$$

$$6y - 9z^2 = -2$$

$$12z - 9x^2 = 4.$$

Find $6x^2 + 9y^2 + 12z^2$.

Problem 13

Find the number of positive integer divisors of 20^{17} that are either perfect squares or perfect cubes.

Problem 14

Let a and b be positive integers such that $a + ab = 1443$ and $ab + b = 1444$. Find $10a + b$.

Problem 15

Find the remainder when 7^{7^7} is divided by 1000.

Problem 16

The set of positive real numbers x that satisfy $2|x^2 - 9| \leq 9|x|$ is an interval $[m, M]$. Find $10m + M$.

Problem 17

Let $a_0, a_1, \dots, a_6$ be real numbers such that $a_0 + a_1 + \dots + a_6 = 1$ and

$$\begin{aligned} a_0 + a_2 + a_3 + a_4 + a_5 + a_6 &= \frac{1}{2} \\ a_0 + a_1 + a_3 + a_4 + a_5 + a_6 &= \frac{2}{3} \\ a_0 + a_1 + a_2 + a_4 + a_5 + a_6 &= \frac{7}{8} \\ a_0 + a_1 + a_2 + a_3 + a_5 + a_6 &= \frac{29}{30} \\ a_0 + a_1 + a_2 + a_3 + a_4 + a_6 &= \frac{143}{144} \\ a_0 + a_1 + a_2 + a_3 + a_4 + a_5 &= \frac{839}{840}. \end{aligned}$$

The value of a_0 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Problem 18

Omar has four fair standard six-sided dice. Omar invented a game where he rolls his four dice over and over again until the number 1 does not appear on the top face of any of the dice. Omar wins the game if on that roll the top faces of his dice show at least one 2 and at least one 5. The probability that Omar wins the game is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Problem 19

Find the sum of all values of $a + b$, where (a, b) is an ordered pair of positive integers and $a^2 + \sqrt{2017 - b^2}$ is a perfect square.

Problem 20

A right circular cone has a height equal to three times its base radius and has volume 1. The cone is inscribed inside a sphere as shown. The volume of the sphere is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

