UW-Whitewater / UT-Dallas Mathematics Meet – April 14, 2005 – MIDDLE SCHOOL © Copyright Titu Andreescu and Jonathan Kane

Problem 1

A cubic inch of the newly discovered material madelbromium weighs 5 ounces. How many pounds will a cubic yard of madelbromium weigh?

Problem 2

Jerry is mowing a rectangular lawn which is 77 feet north to south by 83 feet east to west. His lawn mower cuts a path 18 inches wide. Jerry mows the grass by cutting a path from west to east across the north side of the lawn and then making a right turn cutting a path along the east side of the lawn. When he completes mowing each side of the lawn, he continues by making right turns to mow a path along the next side. How many right turns will he make?

Problem 3



Four rectangular strips each measuring 4 by 16 inches are laid out with two vertical strips crossing two horizontal strips forming a single polygon which looks like a tictack-toe pattern. What is the perimeter of this polygon?

Problem 4

Fill in numbers in the boxes below so that the sum of the entries in each three consecutive boxes is 2005. What is the number that goes into the leftmost box?

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Problem 5

A palindrome is a number that reads the same forwards and backwards such as 3773 or 42924. Find the sum of the twelve smallest five digit palindromes.

Problem 6

We glue together 990 one inch cubes into a 9 by 10 by 11 inch rectangular solid. Then we paint the outside of the solid. How many of the original 990 cubes have just one of their sides painted?

Problem 7

Bill's age is one third larger than Tracy's age. In 30 years Bill's age will be one eighth larger than Tracy's age. How many years old is Bill?

Problem 8

The number 1 is special. The number 2 is special because it is relatively prime to 1. The number 3 is not special because it is not relatively prime to the sum of the special numbers less than it, 1+2. The number 4 is special because it is relatively prime to the sum of the special numbers less than it. So, a number bigger than 1 is special only if it is relatively prime to the sum of the special numbers less than it. Find the twentieth special number.

Problem 9

Find the number of nonnegative integers n for which $(n^2 - 3n + 1)^2 + 1$ is a prime number.

Problem 10

What is the 1000^{th} digit to the right of the decimal point in the decimal representation of $\frac{37}{5500}$?

Problem 11

The work team was working at a rate fast enough to process 1250 items in ten hours. But after working for six hours, the team was given an additional 150 items to process. By what percent does the team need to increase its rate so that it can still complete its work within the ten hours?

Problem 12

Four mathletes and two coaches sit at a circular table. How many distinct arrangements are there of these six people if the two coaches sit opposite each other?

Problem 13

Find x such that
$$\frac{\frac{5}{x-50} + \frac{7}{x+25}}{\frac{2}{x-50} - \frac{3}{x+25}} = 17.$$

Problem 14

Eight identical cubes with of size $1 \times 1 \times 1$ each have the numbers 1 through 6 written on their faces with the number 1 written on the face opposite number 2, number 3 written on the face opposite number 5, and number 4 written on the face opposite number 6. The eight cubes are stacked into a single $2 \times 2 \times 2$ cube. Add all of the numbers appearing on the outer surface of the new cube. Let M be the maximum possible value for this sum, and N be the minimum possible value for this sum. Find M - N.

Problem 15

And it came to pass that Jeb owned over a thousand chickens. So Jeb counted his chickens. And Jeb reported the count to Hannah. And Hannah reported the count to Joshua. And Joshua reported the count to Caleb. And Caleb reported the count to Rachel. But as fate would have it, Jeb had over-counted his chickens by nine chickens. Then Hannah interchanged the last two digits of the count before reporting it to Joshua. And Joshua interchanged the first and the third digits of the number reported to him before reporting it to Caleb. Then Caleb doubled the number reported to him before reporting it to Rachel. Now it so happens that the count reported to Rachel was the correct number of chickens that Jeb owned. How many chickens was that?