PURPLE COMET MATH MEET– April 2008

MIDDLE SCHOOL – PROBLEMS

Problem 1

Find the greatest prime factor of the sum of the two largest two-digit prime numbers.

Problem 2

Starting on April 15, 2008, you can go one day backward and one day forwards to get the dates 14 and 16. If you go 15 days backward and 15 days forward, you get the dates 31 (from March) and 30 (from April). Find the least positive integer k so that if you go k days backward and k days forward you get two calendar dates that are the same.

Problem 3

There were 891 people voting at precinct 91. There were 20 percent more female voters than male voters. How many female voters were there?

Problem 4

While driving his car, Ken pulled off the road to get gasoline when he was $\frac{7}{12}$ of the way through his trip. After driving another eleven miles, he noticed that he was $\frac{13}{20}$ of the way through his trip. How many miles long was his entire trip?

Problem 5

What is the measurement in degrees of the angle formed by the minute and hour hands when a clock reads 12:12?

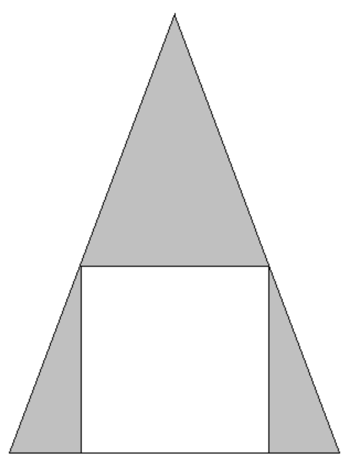
Problem 6

The fraction $\dfrac{2}{\dfrac{2}{\dfrac{2}{\dfrac{2}{2+1}+1}+1}+1+\dfrac{2}{1+\dfrac{2}{1+\dfrac{2}{2+1}}}}$ can be written in the form

$\dfrac{m}{n}$ where m and n are relatively prime positive integers. Find $2m + n$.

Problem 7

The diagram below shows an isosceles triangle with base 21 and height 28. Inscribed in the triangle is a square. Find the area of the shaded region inside the triangle and outside of the square.

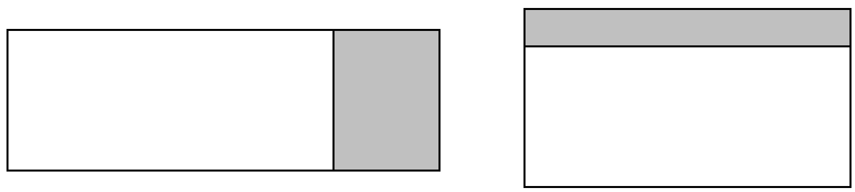


Problem 8

At Mallard High School there are three intermural sports leagues: football, basketball, and baseball. There are 427 students participating in these sports: 128 play on football teams, 291 play on basketball teams, and 318 play on baseball teams. If exactly 36 students participate in all three of the sports, how many students participate in exactly two of the sports?

Problem 9

One container of paint is exactly enough to cover the inside of an old rectangle which is three times as long as it is wide. If we make a new rectangle by shortening the old rectangle by 18 feet and widening it by 8 feet as shown below, one container of paint is also exactly enough to cover the inside of the new rectangle. Find the length in feet of the perimeter of the new rectangle.



Problem 10

A 16×16 square sheet of paper is folded once in half horizontally and once in half vertically to make an 8×8 square. This square is again folded in half twice to make a 4×4 square. This square is folded in half twice to make a 2×2 square. This square is folded in half twice to make a 1×1 square. Finally, a scissor is used to make cuts through both diagonals of all the layers of the 1×1 square. How many pieces of paper result?

Problem 11

When Tim was Jim's age, Kim was twice as old as Jim. When Kim was Tim's age, Jim was 30. When Jim becomes Kim's age, Tim will be 88. When Jim becomes Tim's age, what will be the sum of the ages of Tim, Jim, and Kim?

Problem 12

A city is laid out with a rectangular grid of roads with 10 streets numbered from 1 to 10 running east-west and 16 avenues numbered from 1 to 16 running north-south. All streets end at First and Sixteenth Avenues, and all avenues end at First and Tenth Streets. A rectangular city park is bounded on the north and south by Sixth Street and Eighth Street, and bounded on the east and west by Fourth Avenue and Twelfth Avenue. Although there are no breaks in the roads that bound the park, no road goes through the park. The city paints a crosswalk from every street corner across any adjacent road. Thus, where two roads cross such as at Second Street and Second Avenue, there are four crosswalks painted, while at corners such as First Street and First Avenue, there are only two crosswalks painted. How many crosswalks are there painted on the roads of this city?

Problem 13

Let $A_1 A_2 A_3 \cdots A_{12}$ be a regular dodecagon. Find the number of right triangles whose vertices are in the set $\{A_1, A_2, A_3, \cdots A_{12}\}$.

Problem 14

Ralph is standing along a road which heads straight east. If you go nine miles east, make a left turn, and travel seven miles north, you will find Pamela with her mountain bike. At exactly the same time that Ralph begins running eastward along the road at 6 miles per hour, Pamela begins biking in a straight line at 10 miles per hour. Pamela's direction is chosen so that she will reach a point on the road where Ralph is running at exactly the same time Ralph reaches that same point. Let m and n be relatively prime positive integers such that $\frac{m}{n}$ is the number of hours that it takes Pamela and Ralph to meet. Find $m + n$.

Problem 15

Each of the distinct letters in the following subtraction problem represents a different digit. Find the number represented by the word **TEAM**.

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P U R P L E
 -C O M E T
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    T E A M
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